Sheet six

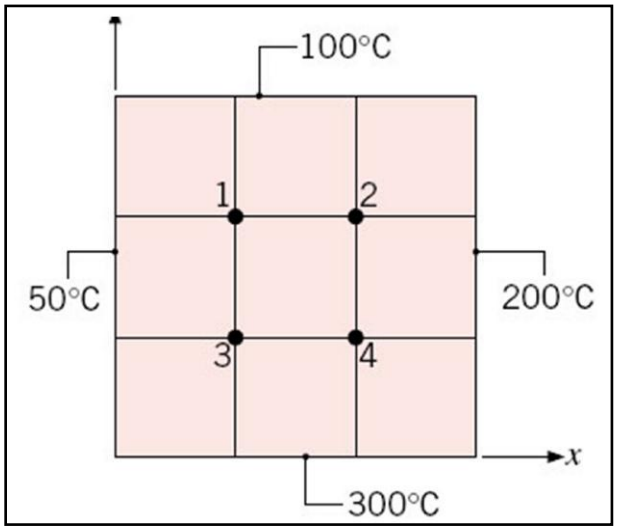
A long solid body with the shown cross sections is subjected to boundary conditions as shown in the following problems where the length is large relative to its cross section dimensions.

Determine

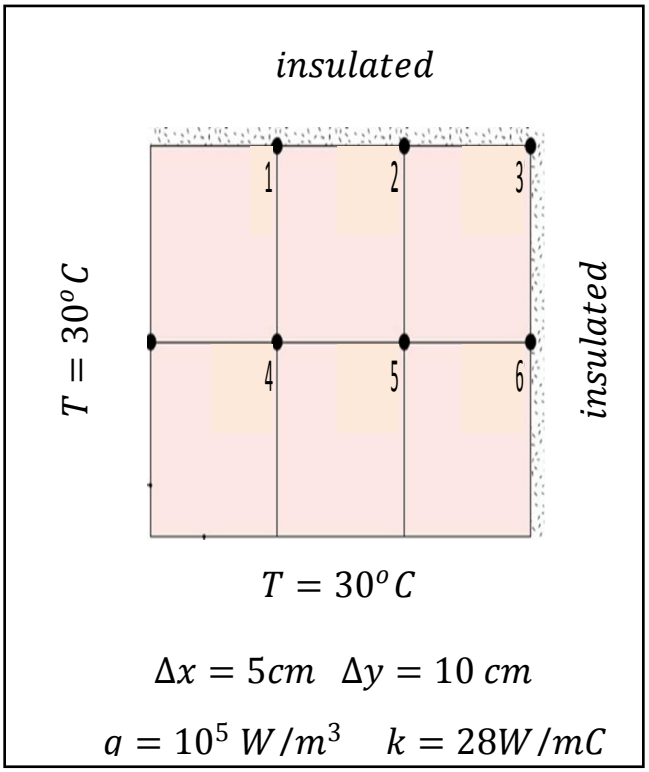
a) The nodal finite-difference equations

b) The nodal temperature using Gauss-Seidel Iteration with maximum relative error of ε and initial value of T_i as shown in following table

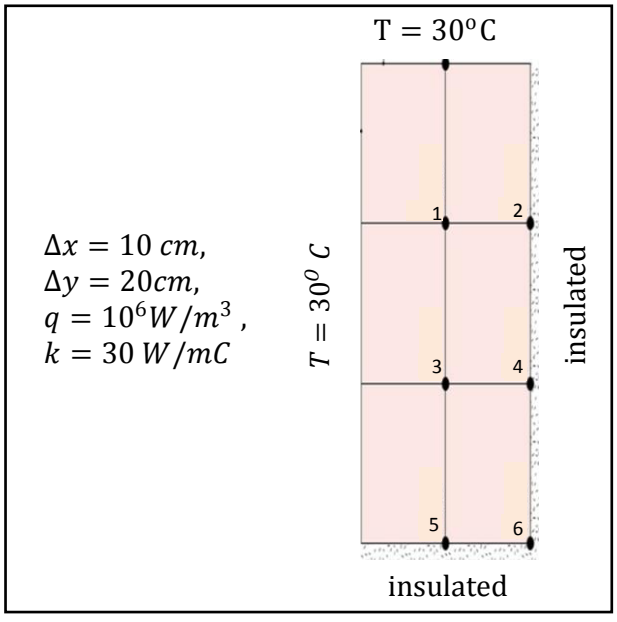
Problem	1	2	3	4	5	6
ε	0.004	0.005	0.007	0.005	0.004	0.007
T_i	150	75	700	450	200	100



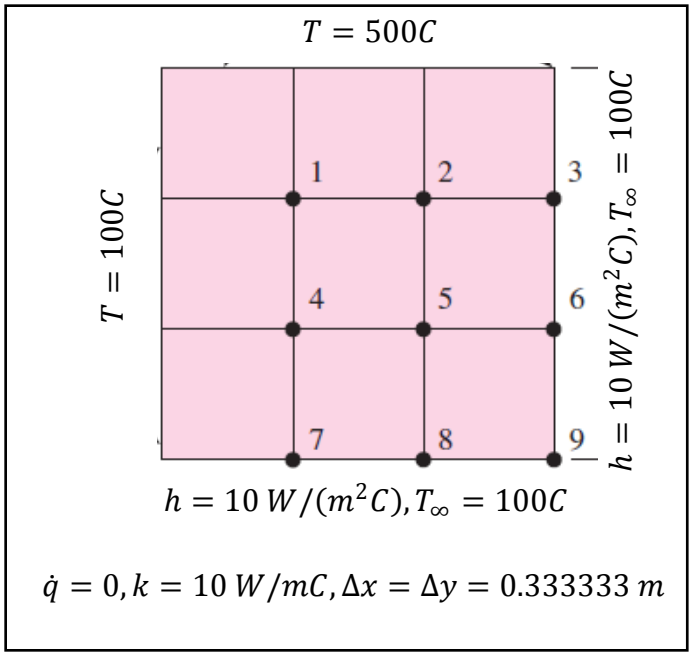
Problem 1



Problem 2

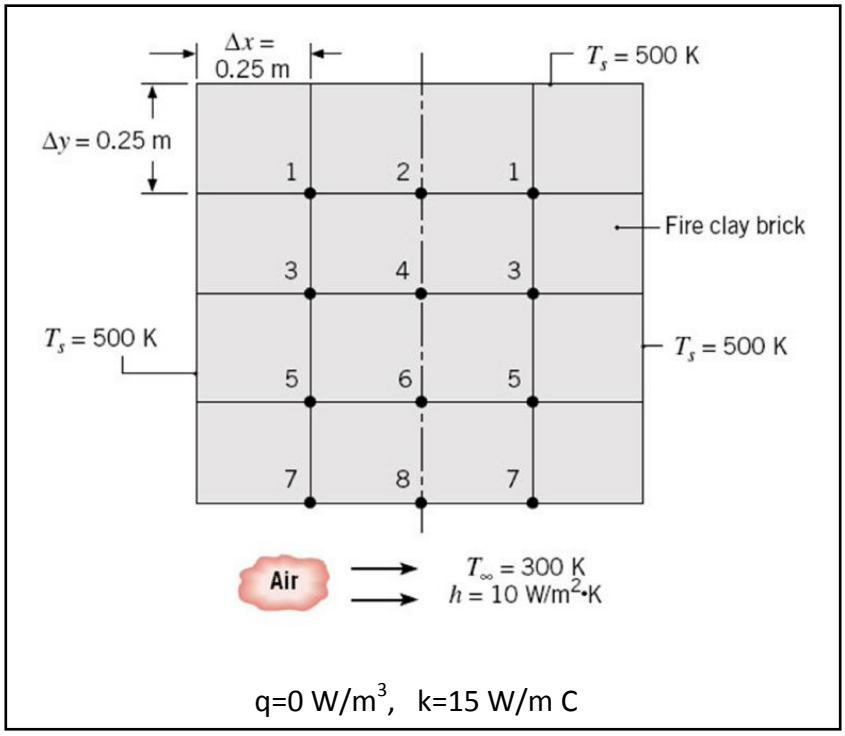


Problem 3



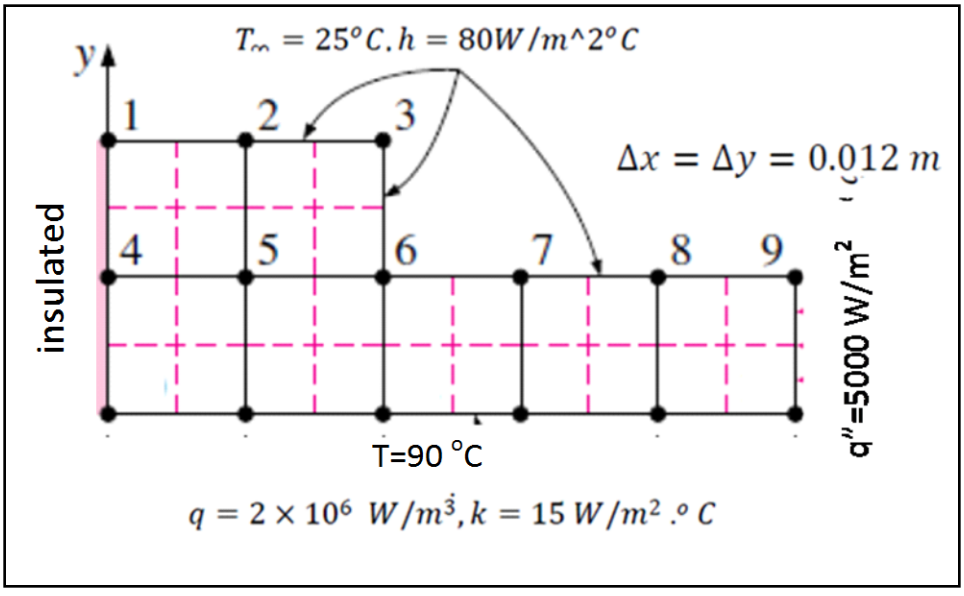
$\dot{q} = 0, k = 10\ W/mC, \Delta x = \Delta y = 0.333333\ m$

Problem 5



q=0 W/m^3, k=15 W/m C

Problem 4



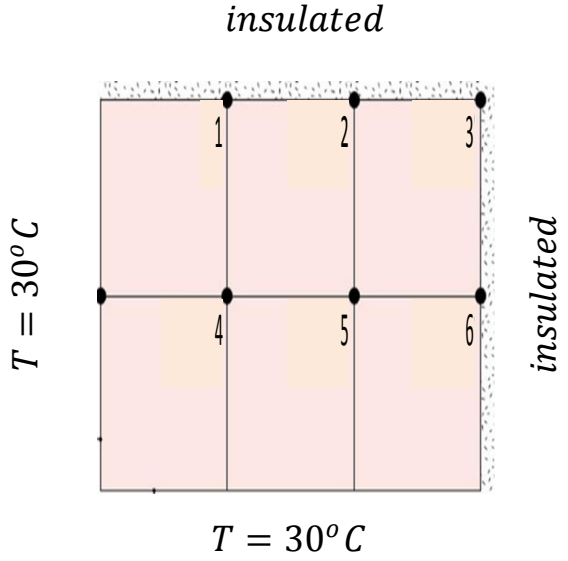
$q = 2 \times 10^6\ W/m^3, k = 15\ W/m^2.°C$

Problem 6

1 - السؤال الثانى والرابع محلولين حل نموذجى (مرفق مع الشيت)
2 - السؤال الاول والثالث سيتم شرحهم فى السكشن
3 - السؤال الخامس والسادس سيحلهم الطالب ويقدمهم فى تقرير منظم فى الموعد الذى سيحدده المعيد
4 - فى حالة تقديم التقرير بعد الموعد المحدد فلن يقبل منه مهما كانت الاعذار ولن توضع له درجة
5 - ارجو الاهتمام بشرح طريقة جاوس-سيدل فى السكاشن لاهميتها فى حل التمارين

حل السؤال الثانى والرابع

2)



$$\Delta x = 5cm \quad \Delta y = 10\ cm$$

$$q = 10^5\ W/m^3 \quad k = 28W/mC$$

a) The nodal finite - difference equations

$node$: 1

$$\frac{\dot{q}\Delta x\Delta y}{2} + k\,\Delta x\,\frac{t_4 - t_1}{\Delta y} + k\,\frac{\Delta y}{2}\,\frac{30 - t_1}{\Delta x} + k\,\Delta\frac{y}{2}\frac{t_2 - t_1}{\Delta x} = 0 \quad \times\frac{2\Delta x}{k\Delta y}$$

$$\frac{\dot{q}(\Delta x)^2}{k} + 2\alpha^2\,t_4 - 2\alpha^2\,t_1 + 30 - t_1 + t_2 - t_1 = 0$$

$$\frac{100000 \times 0.05^2}{28} + 2 \times 0.5^2 \times t_4 + 30 + t_2 - (2 + 2 \times 0.5^2)t_1 = 0$$

$$t_1 = 15.5714 + 0.2t_4 + 0.4t_2$$

$node$: 2

$$\frac{\dot{q}\Delta x\Delta y}{2} + k\,\Delta x\,\frac{t_5 - t_2}{\Delta y} + k\,\Delta\frac{y}{2}\frac{t_3 - t_2}{\Delta x} + k\,\Delta\frac{y}{2}\frac{t_1 - t_2}{\Delta x} = 0 \times\frac{2\Delta x}{k\Delta y}$$

$$\frac{\dot{q}(\Delta x)^2}{k} + 2\alpha^2\,t_5 - 2\alpha^2\,t_2 + t_3 - t_2 + t_1 - t_2 = 0$$

$$\frac{100000 \times 0.05^2}{28} + 2 \times 0.5^2 \times t_5 + t_3 + t_1 - (2 + 2 \times 0.5^2)t_2 = 0$$

$$t_2 = 3.57143 + 0.2t_5 + 0.4t_3 + 0.4t_1$$

$node$: 3

$$\frac{\dot{q}\Delta x\Delta y}{4} + k\,\frac{\Delta y}{2}\frac{t_2 - t_3}{\Delta x} + k\,\frac{\Delta x}{2}\frac{t_6 - t_3}{\Delta y} = 0 \quad \times\frac{2\Delta x}{k\Delta y}$$

$$\frac{\dot{q}\Delta x^2}{2k} + t_2 - t_3 + \alpha^2 t_6 - \alpha^2 t_3 = 0$$

$$\frac{100000 \times 0.05^2}{2 \times 28} + t_2 + 0.5^2 t_6 - (1 + 0.5^2) t_3 = 0$$

$$t_3 = 3.57143 + 0.8t_2 + 0.2t_6$$

$node$: 4

$$q\,\Delta x\Delta y + k\,\Delta y\,\frac{30 - t_4}{\Delta x} + k\,\Delta y\,\frac{t_5 - t_4}{\Delta x} + k\,\Delta x\,\frac{t_1 - t_4}{\Delta y} + k\,\Delta x\,\frac{30 - t_4}{\Delta y} = 0 \quad \times\frac{\Delta x}{k\Delta y}$$

$$\frac{q(\Delta x)^2}{k} + 30 - t_4 + t_5 - t_4 + \alpha^2 t_1 - \alpha^2 t_4 + \alpha^2 \times 30 - \alpha^2 t_4 = 0$$

$$\frac{100000 \times 0.05^2}{28} + 30 + t_5 + 0.5^2 t_1 + 0.5^2 \times 30 - (2 \times 0.5^2 + 2)t_4 = 0$$

$$t_4 = 27.5714 + 0.4t_5 + 0.1t_1$$

$node$: 5

$$q\,\Delta x\Delta y + k\,\Delta y\,\frac{t_4 - t_5}{\Delta x} + k\,\Delta y\,\frac{t_6 - t_5}{\Delta x} + k\,\Delta x\,\frac{t_2 - t_5}{\Delta y} + k\,\Delta x\,\frac{30 - t_5}{\Delta y} = 0 \quad \times\frac{\Delta x}{k\Delta y}$$

$$\frac{q(\Delta x)^2}{k} + t_4 - t_5 + t_6 - t_5 + \alpha^2 t_2 - \alpha^2 t_5 + \alpha^2 \times 30 - \alpha^2 t_5 = 0$$

$$\frac{100000 \times 0.05^2}{28} + t_4 + t_6 + 0.5^2 t_2 + 0.5^2 \times 30 - (2 \times 0.5^2 + 2)t_5 = 0$$

$$t_5 = 15.5714 + 0.4t_4 + 0.4t_6 + 0.1t_2 +$$

$node$: 6

$$\frac{\dot{q}\Delta x\Delta y}{2} + k\,\Delta y\,\frac{t_5 - t_6}{\Delta x} + k\,\frac{\Delta x}{2}\,\frac{30 - t_6}{\Delta y} + k\,\frac{\Delta x}{2}\frac{t_3 - t_6}{\Delta y} = 0 \quad \times\frac{2\Delta x}{k\Delta y}$$

$$\frac{\dot{q}(\Delta x)^2}{k} + 2t_5 - 2t_6 + \alpha^2 \times 30 - \alpha^2 t_6 + \alpha^2 t_3 - \alpha^2 t_6 = 0$$

$$\frac{100000 \times 0.05^2}{28} + 2t_5 + 0.5^2 \times 30 + 0.5^2 \times t_3 - (2 + 2 \times 0.5^2)t_6 = 0$$

$$t_6 = 6.57143 + 0.1t_3 + 0.8t_5$$

b) The nodal temperature using Gauss-Seidel Iteration with relative error of 0.005and initial value of 75.

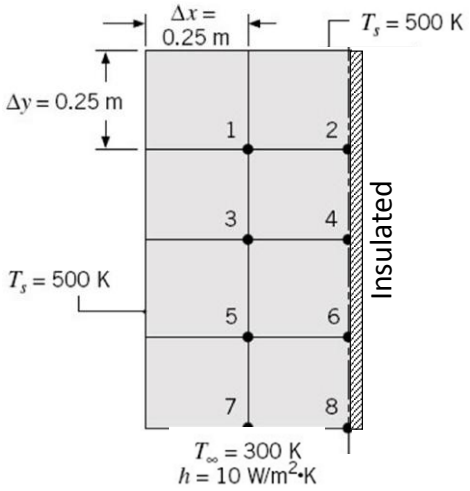
$$t_1 = 15.5714 + 0.2t_4 + 0.4t_2$$
$$t_2 = 3.57143 + 0.2t_5 + 0.4t_3 + 0.4t_1$$
$$t_3 = 3.57143 + 0.8t_2 + 0.2t_6$$
$$t_4 = 27.5714 + 0.4t_5 + 0.1t_1$$
$$t_5 = 15.5714 + 0.4t_4 + 0.4t_6 + 0.1t_2 +$$
$$t_6 = 6.57143 + 0.8t_5 + 0.1t_3$$

$$error = \left| \frac{t_{new} - t_{old}}{t_{old}} \right|$$

i	0	1	error	2	error	3	error	4	error	5	error	6	error
T1	75	60.571	0.1924	57.417	0.0521	57.668	0.0044	57.977	0.0054	58.234	0.0044	58.427	0.0033
T2	75	72.8		72.923		73.449	0.0072	73.949		74.337	0.0053	74.62	0.0038
T3	75	76.811		77.289		77.907		78.422		78.807		79.082	0.0035
T4	75	63.629		64.634		65.128		65.417		65.601		65.725	0.0019
T5	75	78.303		79.475		80.12		80.516		80.776		80.953	0.0022
T6	75	76.895		77.881		78.458		78.827		79.073		79.242	0.0021

4)

يلاحظ فى هذ السؤال انه يوجد تماثل حول المحور الراسى فلذلك يتم حل نصف الشكل مع اعتبار خطوط التماثل معزوله



q=0 W/m^3 , k=15 W/m C

a) The nodal finite - difference equations

$node$: 1

$$k\ l\ \frac{500\ -\ t_1}{l} +\ \ k\ l\ \frac{t_2\ -\ t_1}{l} +\ \ k\ l\ \frac{500\ -\ t_1}{l} +\ \ k\ l\ \frac{t_3\ -\ t_1}{l} =\ 0\ \ \times \frac{1}{k}$$

$$500\ -\ t_1 + t_2 - t_1 + 500\ -\ t_1 + t_3 - t_1 = 0$$

$$500\ +\ t_2 + 500\ +\ t_3 - 4\,t_1 = 0$$

$$t_1 = 250 + 0.25t_2 +\ 0.25t_3$$

$node$: 2

$$k\ l\ \frac{t_1 - t_2}{l} + k\ \frac{l}{2}\frac{t_4 - t_2}{l} + k\ \frac{l}{2}\frac{500 - t_2}{l} = 0 \quad \times \frac{2}{k}$$

$$2t_1 - 2t_2 +\ t_4 -\ t_2 + 500 -\ t_2 = 0$$

$$2t_1 + t_4 + 500 - (4)t_2 = 0$$

$$t_2 = 125 + 0.25t_4 +\ 0.5t_1$$

$node$: 3

$$k\ l\ \frac{500\ -\ t_3}{l}+\ \ k\ l\ \frac{t_4\ -\ t_3}{l}+\ \ k\ l\ \frac{t_1\ -\ t_3}{l}+\ \ k\ l\ \frac{t_5\ -\ t_3}{l}\ =\ 0\ \ \times\frac{1}{k}$$

$$500\ -\ t_3\ +\ t_4\ -\ t_3\ +\ t_1\ -\ t_3\ +\ t_5\ -\ t_3\ =\ 0$$

$$500\ +\ t_4\ +\ t_1\ +\ t_5\ -\ 4\,t_3\ =\ 0$$

$$t_3 = 125 + 0.25t_4\ +\ \ 0.25t_1\ +\ \ 0.25t_5$$

$node$: 4

$$k\ \ l\ \frac{t_3 - t_4}{l} + k\ \frac{l}{2}\frac{t_6 - t_4}{l} + k\ \frac{l}{2}\frac{t_2 - t_4}{l} = 0\quad \times\frac{2}{k}$$

$$2t_3 - 2t_4 +\ \ t_6 -\ \ t_4 +\ \ t_2 -\ \ t_4 = 0$$

$$2t_3 + t_6 + t_2 - (4)t_4 = 0$$

$$t_4 = 0.25t_2\ +\ \ 0.25t_6\ +\ \ 0.5t_3$$

$node$: 5

$$k\ \ l\ \frac{500\ -\ t_5}{l}+\ \ k\ \ l\ \frac{t_6\ -\ t_5}{l}+\ \ k\ \ l\ \frac{t_3\ -\ t_5}{l}+\ \ k\ \ l\ \frac{t_7\ -\ t_5}{l}\ =\ 0\ \ \times\frac{1}{k}$$

$$500\ -\ t_5\ +\ t_6\ -\ t_5\ +\ t_3\ -\ t_5\ +\ t_7\ -\ t_5\ =\ 0$$

$$500\ +\ t_6\ +\ t_3\ +\ t_7\ -\ 4\,t_5\ =\ 0$$

$$t_5 = 125 + 0.25t_6\ +\ \ 0.25t_3\ +\ \ 0.25t_7$$

$node$: 6

$$k\ \ l\ \frac{t_5 - t_6}{l} + k\ \frac{l}{2}\frac{t_8 - t_6}{l} + k\ \frac{l}{2}\frac{t_4 - t_6}{l} = 0\quad \times\frac{2}{k}$$

$$2t_5 - 2t_6 +\ \ t_8 -\ \ t_6 +\ \ t_4 -\ \ t_6 = 0$$

$$2t_5 + t_8 + t_4 - (4)t_6 = 0$$

$$t_6 = 0.25t_4\ +\ \ 0.25t_8\ +\ \ 0.5t_5$$

$node$: 7

$$h\ \ l\ \ (t_\infty - t_7\ ) + k\ \ l\ \frac{t_5 - t_7}{l}\ +\ \ k\ \frac{l}{2}\frac{t_8 - t_7}{l}\ +\ \ k\ \frac{l}{2}\frac{500 - t_7}{l}\ =\ \ 0\quad \times\frac{2}{k}$$

$$\frac{2\ \ hlt_\infty}{k} - 2\ \frac{hl}{k}\ t_7 + 2\ \ t_5 - 2\ \ t_7 + t_8 - t_7 + 500 - t_7 = 0$$

$$\frac{2\times\ 10\times\ 0.25\times\ 300}{15} + 2t_5 + t_8 + 500 - \left(2\ \ \times\ 10\times\frac{0.25}{15} + 4\ \right)t_7 = 0$$

$$t_7 = 138.462 + 0.461538t_5\ +\ \ 0.230769t_8$$

$node$: 8

$$h_x\,\frac{l}{2}\,\left(t_{\infty,x}\ -\ \ t_8\right) + k\ \frac{l}{2}\frac{t_7 - t_8}{l}\ +\ \ k\ \frac{l}{2}\frac{t_6 - t_8}{l} = 0\ \ \times\frac{2}{k}$$

$$\frac{h_x\ lt_{\infty,x}}{k} - \frac{h_x\ l}{k}\ t_8 +\ \ t_7 -\ \ t_8\ +\ \ \ t_6 -\ \ t_8\ = 0$$

$$\frac{10\times\ 0.25\times\ 300}{15}\ +\ \ \ t_7\ +\ \ \ t_6\ -\ \left(\frac{10\times\ 0.25}{15} +\ \ 2\ \right)\ t_8\ =\ \ 0$$

$$t_8 = 23.0769 + 0.461538t_7\ +\ \ 0.461538t_6$$

b) The nodal temperature using Gauss-Seidel Iteration with relative error of 0.005 and initial value of 450

$t_1 = 250 + 0.25t_2 + 0.25t_3$
$t_2 = 125 + 0.25t_4 + 0.5t_1$
$t_3 = 125 + 0.25t_4 + 0.25t_1 + 0.25t_5$
$t_4 = 0.25t_2 + 0.25t_6 + 0.5t_3$
$t_5 = 125 + 0.25t_6 + 0.25t_3 + 0.25t_7$
$t_6 = 0.25t_4 + 0.25t_8 + 0.5t_5$
$t_7 = 138.462 + 0.461538t_5 + 0.230769t_8$
$t_8 = 23.0769 + 0.461538t_7 + 0.461538t_6$

$$error = \left|\frac{t_{new} - t_{old}}{t_{old}}\right|$$

i	0	1	error	2	error	3	error	4	error	5	error
T1	450	475	0.0556	485.938	0.023	491.016	0.0105	493.825	0.0057	495.319	0.003
T2	450	475		484.375		489.648		492.511		494.103	0.0032
T3	450	468.75		479.688		485.652		488.764		490.625	0.0038
T4	450	465.625		476.563		482.395		485.775		487.843	0.0043
T5	450	467.188		475.03		478.837		481.408		483.101	0.0035
T6	450	462.5		468.629		473.059		476.017		477.947	0.0041
T7	450	457.933		461.066		463.809		465.76		467.064	0.0028
T8	450	447.892		452.167		455.478		457.743		459.236	0.0033